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Some Aspects of the Applications of Hybrid Propulsion Systems

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HYBRID propulsion systems using solid fuels and liquid oxidizers possess possibilities for high performance with simplicity and resulting reliability. Unclassified chemical data indicate that some hybrid propellants will provide theoretical vacuum specific impulse values of about 500 lb-sec/lb. When reasonable combustion and nozzle efficiencies are applied to this theoretical value, the potential delivered performance is still very high. Most effort to date has been in the areas of experimental and theoretical work on the burning mechanism and the performance for various propellant combinations. Consideration must also be given to the application of hybrids to the specific requirements of a vehicle system. Several aspects of hybrid propulsion system applications come to light when quantitative mission requirements and related vehicles are analyzed. Two of the more interesting aspects, namely, the requirement for achieving constant thrust and wide throttling, are the subjects of this note.

Regression Rate

Hybrid combustion is dependent on the liquid oxidizer flow and the solid fuel burning or regression rate. The regression-rate expression that relates the pertinent ballistic parameters is of the general form:

$$r = BG_T^x/L_p^y + C$$

where r is the regression rate, L_p is the grain port length B is the convection heat-transfer constant, y is the grain port length exponent, x is the mass flux exponent, C is the constant, and G_T is the total mass flow rate per square inch of grain port area (mass flux).

This expression can be derived explicitly for different propellant combinations and fuel grain configurations to fit available experimental and theoretical data. Its empirically determined constants are influenced by the propellant combination and the temperature at combustion conditions. To illustrate how the various parameters affect the O/F ratio (oxidizer-to-fuel weight ratio), regression rate, and total weight flow, the general regression-rate expression was made explicit for a circular port grain. Assuming a constant oxidizer flow rate, specific values for x and y, and ignoring the constant C, the O/F varies with the relationship $K_1 = (O/F)(O/F + 1)^4L_p^4/D_p^3$; the regression rate varies with the relationship $r = K_2(O/F + 1)^4L_p^3/D_p^4$; and the total flow rate (and therefore thrust) varies with the expression $\dot{W}_T = K_3(O/F + 1)^5L_p^4/D_p^3$, where D_p is the fuel grain port diameter.

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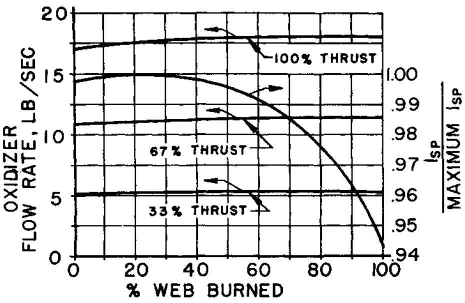

Fig. 1 Oxidizer flow rate and specific impulse.

Constant Thrust

The preceding expressions indicate that, as the port diameter increases during burning, the only way to maintain constant thrust is to vary the O/F ratio. Control of the O/F ratio is accomplished by varying the oxidizer flow rate. Figure 1 shows the variation in oxidizer flow rate required to achieve constant thrust for a hypothetical design and the resultant variation in specific impulse. The theoretical change in I_{sp} with O/F and P_c can be seen from Fig. 2, which is typical of the ballistic performance of all rocket systems. This requirement to vary the oxidizer flow rate somewhat complicates an otherwise extremely simple system. We therefore will examine the consequences of operating the motor with a constant oxidizer flow rate, which reflects the minimum complexity system when an absolutely constant thrust level is not required. The burning time is under 200 sec, and no throttling is specified. Restart capability is not discussed in this note, although it is easily accomplished by the use of a hypergolic hybrid propellant combination.

Several possible grain designs can be considered for the hybrid engine. Since the star design permits a smaller length-to-diameter ratio than does the circular port and is generally simpler than the other types, it was selected for the analysis of the effect of constant oxidizer flow rate on thrust and I_{sp}. The simple internal ballistic equations are only slightly more complex with respect to the geometrical parameters. A constant perimeter, eight-point star configuration with constant oxidizer flow rate produces a thrust that varies less than 12% (Fig. 3). The average I_{sp} for the total burning time varies less than 0.3% from the nominal value at optimum O/F. Although the O/F ratio varies, complete (within reasonable expulsion and sliver tolerances) propellant utilization poses no problem because the proper amounts of both fuel and oxidizer can be predetermined and the propulsion system loaded accordingly.

Wide Throttling

A hybrid engine capable of 50/1 throttling represents the other end of design complexity. With oxidizer injected at the head end only, 50/1 throttling would require varying

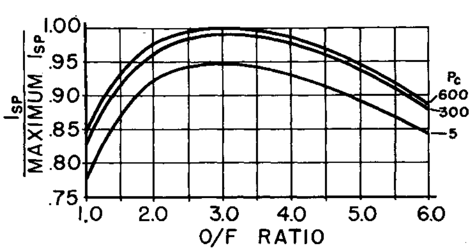

Fig. 2 Theoretical kinetic specific impulse.

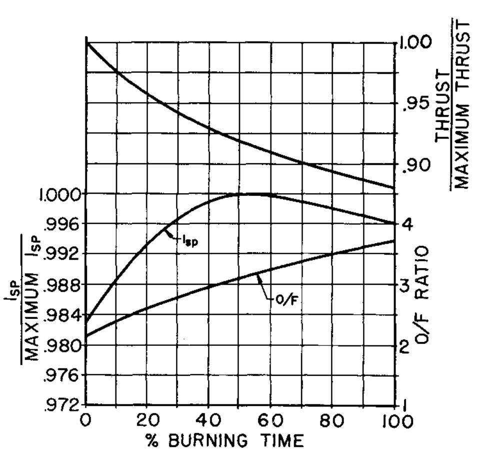

Fig. 3 Motor performance.

oxidizer injection rates, resulting in O/F ratios of about 1.2 to 3.8 at minimum and maximum thrust, respectively. Referring again to Fig. 2, it is easy to see the reduction in specific impulse at the low O/F ratios and reduced chamber pressure. However, the major effect of this wide O/F range is in propellant utilization. With a fixed amount of propellant provided, 75% of which is oxidizer, continuous low-thrust operation at less than the design O/F would result in an excess of oxidizer; e.g., 61% of the oxidizer would remain after operation at $O/F = 1.2$. Continuous high-thrust operation would not utilize all of the fuel available. Therefore, any thrust level other than that corresponding to near optimum O/F ratio reduces effective mass fraction as well as the specific impulse of the propulsion system. For the vehicle assumed in this analysis, the accumulated effect of performance degradation on velocity is shown in Fig. 4 as a function of throttling ratio.

Operation at a constant O/F ratio at or near optimum can be achieved by utilizing aft-end make up in the hybrid motor design (Fig. 5). Oxidizer injectors are provided at both ends of the chamber. At the head end, sufficient oxidizer is sprayed into the fuel-lined chamber to produce the required fuel-weight flow rate for any selected thrust level. The remaining oxidizer to provide the optimum mixture ratio is injected at the aft end of the fuel grain just upstream of the nozzle. The use of this technique permits maintenance of an optimum O/F ratio at any thrust level. A better understanding of what occurs is obtained by examining the equations presented earlier. A low level of oxidizer flow is required to provide a low level of thrust. A low chamber pressure also results, but this affects only the I_{sp} and not the regression rate. The oxidizer injected aft of the solid fuel grain has no effect on the fuel flow and thereby only increases the O/F ratio. Proper liquid oxidizer flow control then can result in optimum O/F ratio operation. A single flow-control valve can be utilized to provide the proper amounts of oxidizer flow to the injectors at each end of the engine. The relative total oxidizer flow rate and the oxidizer flow rate at each end are shown in Fig. 4 as functions of the percentage of thrust desired for a typical design. The aft-end make-up technique is considered to be a practical solution to the problems involved in wide throttling of a hybrid engine.

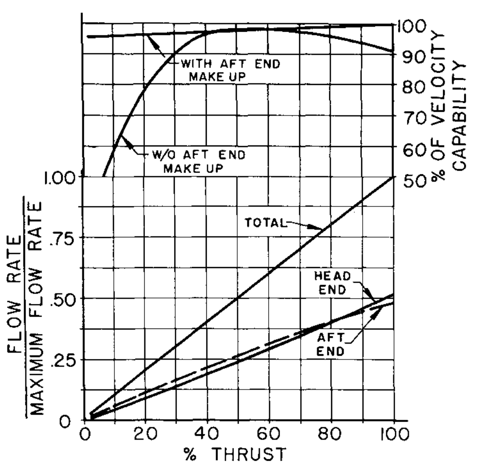

Fig. 4 Vehicle performance and flow rates.

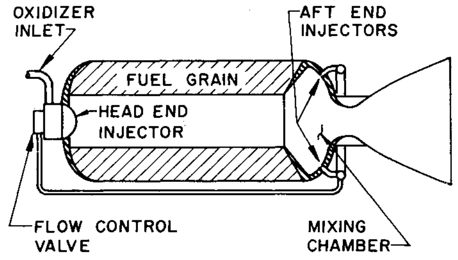

Fig. 5 Hybrid engine schematic.

Conclusions

Achieving constant thrust in a hybrid engine requires a varying oxidizer flow rate and results in some degradation in performance due to degraded specific impulse. Nearly constant thrust is easier to achieve through the use of a constant oxidizer flow rate and with only slight performance penalties. Wide throttling requires a unique design solution for both thrust control and maximum propellant utilization. The aft-end make-up technique provides this solution.

Liquid Frequencies and Damping in Compartmented Cylindrical Tanks

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DEMANDS for increasing the payload in space missions have led to larger and longer launch vehicles and hence to fuel and oxidizer tanks of enormous capacity and, unfortunately, very low propellant slosh frequencies. Such low frequencies are undesirable because of the likelihood of coupling with structural modes or the autopilot system. Clustering smaller diameter tanks has offered a means of providing large volume with substantially higher liquid resonant frequencies than in a single tank for a vehicle with the same over-all fineness ratio. Compartmentation of a single cylindrical tank into sectors also provides a means of shifting the liquid resonant frequencies.

The present note presents data on liquid resonant frequencies and damping coefficients in a compartmented tank undergoing forced translational oscillation. Flat-bottomed cylindrical tanks were compartmented into equal sectors of 90°, 60°, or 45°. Materials for the sector walls varied from solid to perforated stocks with various hole diameters and

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